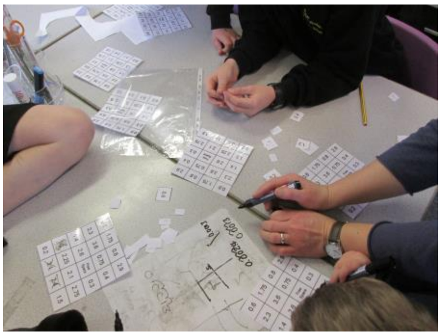
Steeple Morden C of E Primary School

A Parent's Guide:-

How do we teach calculation in school?





'Maths Open Morning' – Friday 2nd February 2018

Maths is all around us…. we teach it in a whole range of ways but aim to develop confidence from the youngest age and the belief that Maths can be fun!

The teaching of Mathematics has changed significantly in recent times.

Fluency in calculation is of course only one facet of our Maths teaching in school.

Developing **reasoning** and **problem solving** skills are equally important.

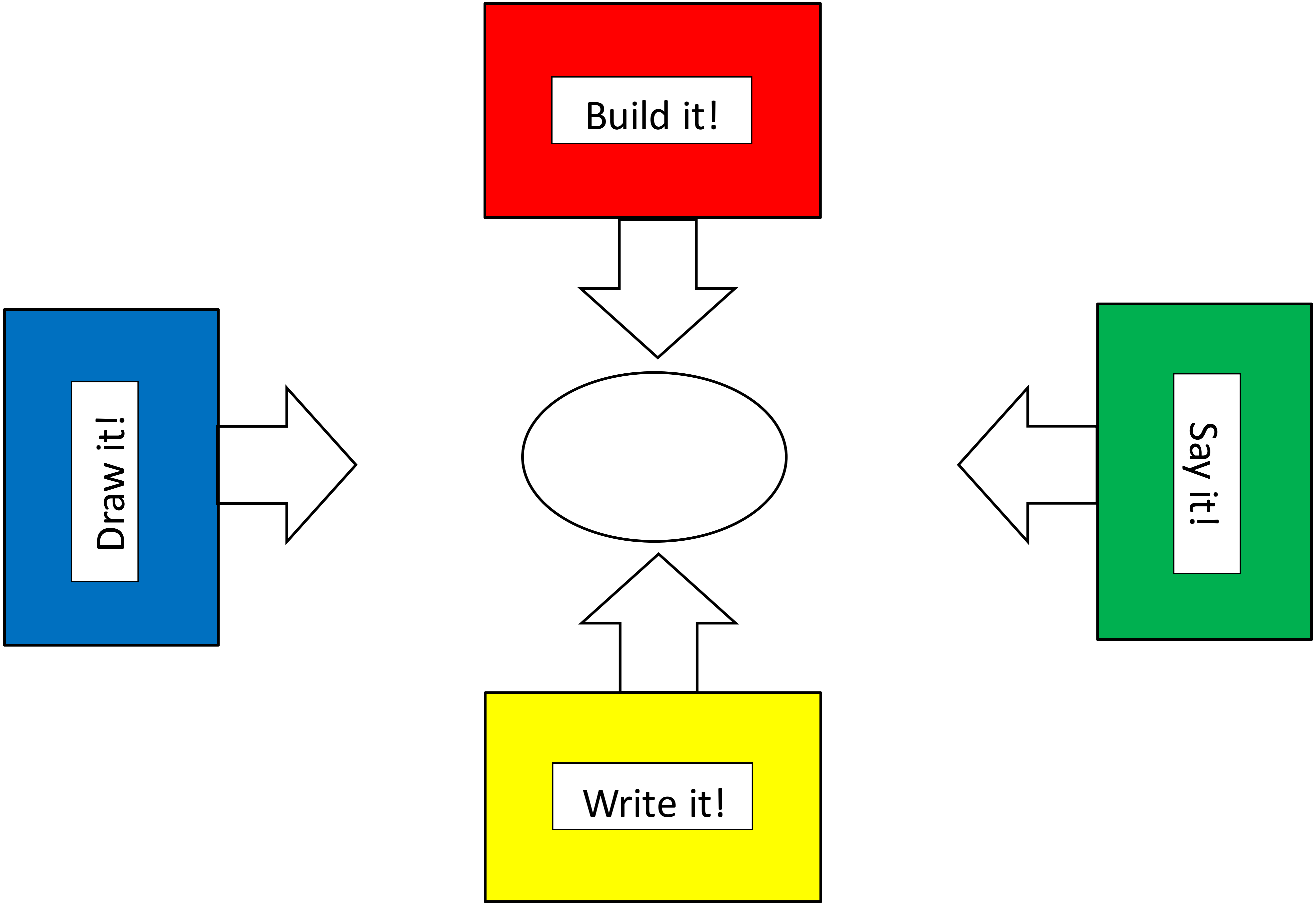
This booklet is intended to be used as a guide to how we teach calculation methods in school using a range of mental Maths strategies, jottings and eventually more formal written methods.

Whether or not children can complete a page of 'sums' or number sentences is not necessarily a measure of their mathematical understanding. Before any written methods of calculation are taught, children need to have used a whole range of practical 'concrete' resources to build key skills to enable them to work out calculations mentally. This begins in Early Years and continues into Year 6 with our class counting boxes.

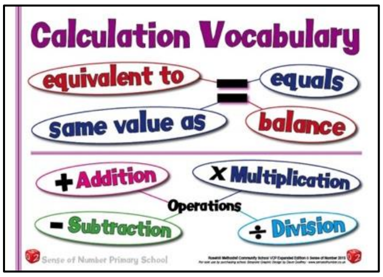
'Maths Talk' is extremely valuable - using mathematical vocabulary daily will enable children to develop their understanding of mathematical language as they move through school.

There is no one 'correct' method of getting to an answer – children may navigate their way to an answer in several different ways – we talk about all of these methods and equip children with a range of methods to tackle each mathematical problem that they may face!

By the time children reach Years 5 and 6, they will be equipped to decide when it is best to use a mental method, an informal jotting or a more formal written method based on the knowledge that they are in control of this choice as they are able to carry out all methods with confidence.

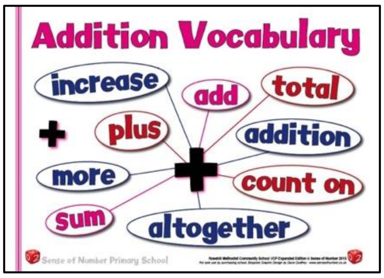

Build it!
Draw it!
Say it!
Write it!

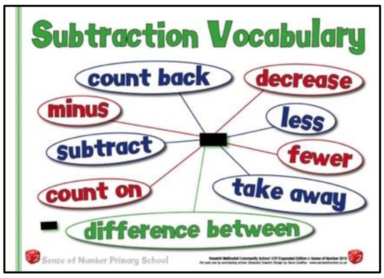
The Importance of Vocabulary in Calculation

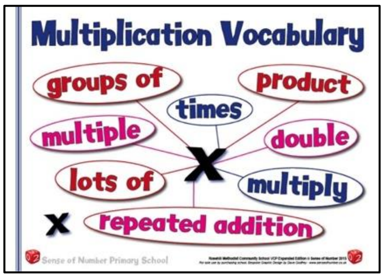


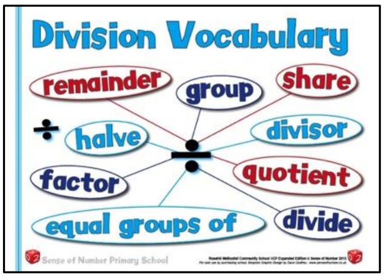
Key Vocabulary:

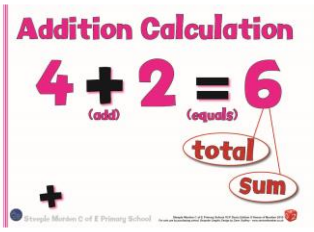
It is important that children are introduced to the calculation vocabulary that they will continue to build upon in school.

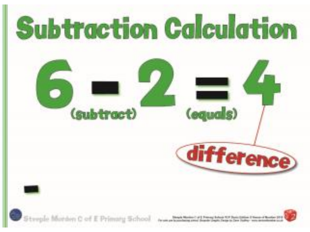


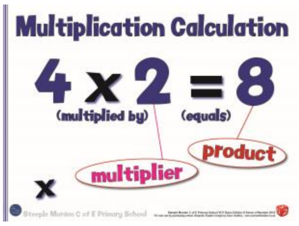


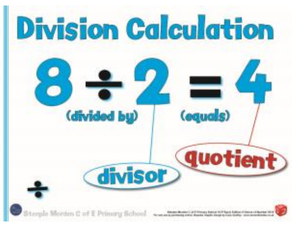




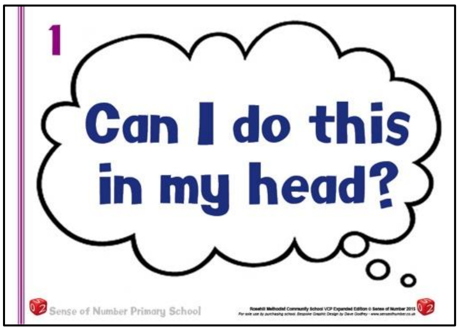








Mental Methods of Calculation



Oral and mental work in Mathematics is essential, particularly so in calculation. When attempting to work out any calculation, the first question a child should ask is always, 'can I work this out in my head?'

The ability to calculate mentally forms the basis of all methods of calculation. A good knowledge of numbers or a **'sense' of number**' takes practice and repetition of key skills.

Here are some examples:-

<u>Recall key number facts instantly</u>, eg, number bonds to 20, and doubles of all numbers up to double 20 (Year 2) and multiplication facts up to 12 × 12 (Year 4);

<u>Use taught strategies to work out the calculation</u>, eg, recognise that addition can be done in any order and use this to add mentally a one-digit number to a one-digit or two-digit number (Year 1), add two-digit numbers in different ways (Year 2), add and subtract numbers mentally with increasingly large numbers (Year 5);

<u>Understand how the rules and laws of arithmetic are used and applied</u> – for example to use commutativity in multiplication (Year 2), estimate the answer to a calculation and use inverse operations to check answers (Years 3 & 4), use knowledge of the order of operations to carry out calculations involving the four operations (Year 6).

The first 'answer' that a child may give to a mental calculation question would be based on instant recall. eg, "What is 12 + 4?", "What is 12 x 4?", "What is 12 – 4?" or "What is 12÷ 4?" giving the immediate answers "16", "48", "8" or "3"
Other children would still work these calculations out mentally by counting on from 12 to 16, counting in 4s to 48, counting back in ones to 8 or counting up in 4s to 12.

The following pages give more detail about the methods of calculation as they are introduced in different year groups.

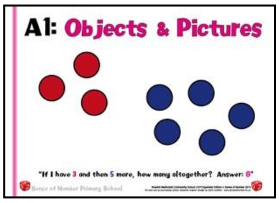
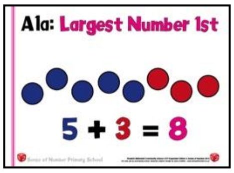
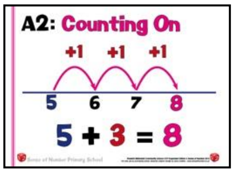
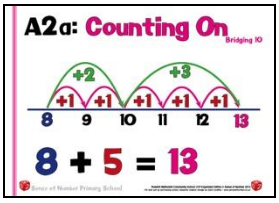
Addition Progression

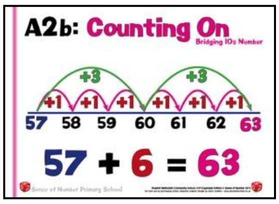
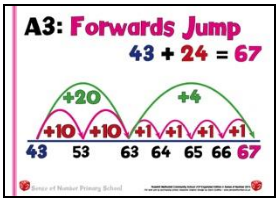
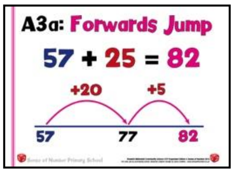
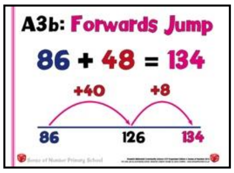
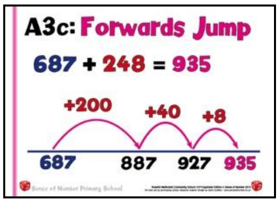
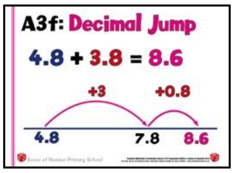
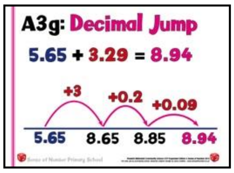
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

Children need to acquire one efficient written method of calculation for addition that they know they can rely on when mental methods are not appropriate.

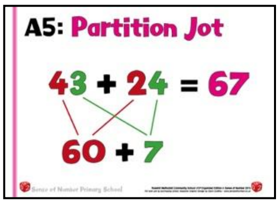
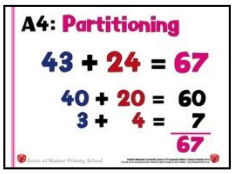
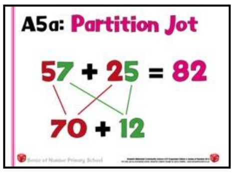
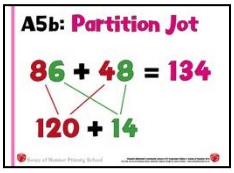
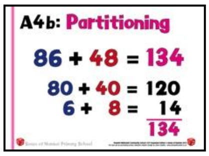
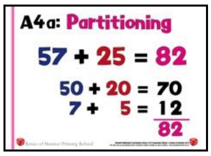
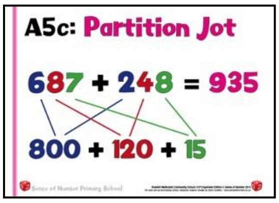
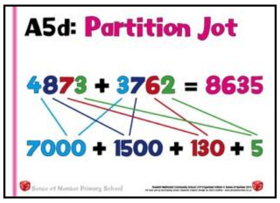
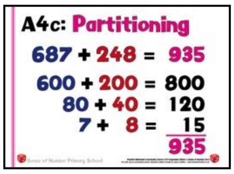
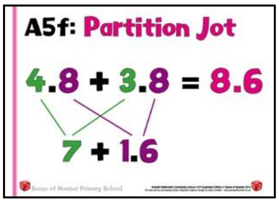
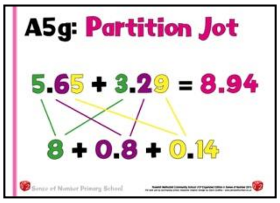
To add successfully, children need to be able to:

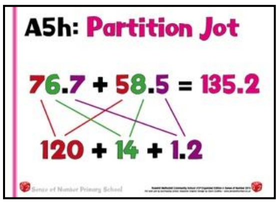
- recall all addition pairs to 9 + 9 and complements in 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

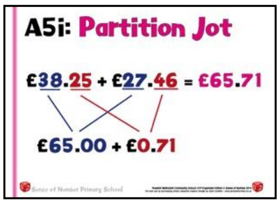
Stage 1	**Finding a Total and the Empty Number Line**	**Alternative Method: Counting on Mentally or as a jotting**
FS/Y1	Initially, children need to represent addition using a range of different resources, and understand that a total can be found by counting out the first number, counting out the second number then counting how many there are altogether. **3 + 5 = 8**	
		3 (held in head) then use fingers to count on 5 ("3… 4,5,6,7,8)
	This will quickly develop into placing the largest number first, either as a pictorial / visual method or by using a number line. **5 + 3 = 8**	
	 	5 (held in head) then count on 3 ("5 … 6, 7, **8**")
Y1/2	Steps in addition can be recorded on a number line. The steps often bridge through 10. **8 + 5 = 13**	
		8 (held in head) then count on 5 ("8 … 9, 10, 11, 12, **13**")
	The next step is to bridge through a multiple of 10.	

		57 (held in head) then count on 6 ("57 … 58,59,60,61,62,63")
	The number line becomes a key image for demonstrating how to keep one number whole, whilst partitioning the other number. Teach the children firstly to add the tens then the ones individually (43 + 24 = 43 + 10 + 10 + 1 + 1 + 1 + 1) before progressing to counting on in tens and ones (43 + 20 + 4)	This method will be a jotting approach, and may look like the following examples: - 43 + 24 43 + 20 = 63 63 + 4 = 67
		Or 43 + 20 + 4 = 67
	Develop to crossing the 10s, then the 100s boundary 57 + 25 = 82 86 + 48 = 134	
	 	57 + 25 57 + 20 = 77 77 + 5 = 82 57 + 20 + 5 = 82 86 + 48 86 + 40 = 126 126 + 8 = 134 86 + 40 + 8 = 1
Y3/4	For some children, this method can still be used for 3 digit calculations	**Number lines support children's thinking if they find partitioning / column addition difficult, as it simply involves counting on in 100s, 10s & 1s.**
		687 + 248 687 + 200 = 887 887 + 40 = 927 927 + 8 = 935 or 687 + 200 + 40 + 8 = 935
Y5/6	In Years 5 and 6, if necessary, children can return to this method to support their understanding of decimal calculation	
	 	4.8 + 3.8 4.8 + 3 = 7.8 7.8 + 0.8 = 8.6 Or 4.8 + 3 + 0.8 = 8.6

Hopefully, with the above calculation, many children would mentally Round & Adjust (4.8 + 4 – 0.2 = 8.6)

Stage 2	Partition Jot	Alternative Method: Traditional Partitioning
Y2/3	**Traditionally, partitioning has been presented using the method on the right. This can be refined to the 'partition jot' method which will improve speed and consistency for mental to written (or written to mental) progression.**	Record steps in addition using partition, initially as a jotting: - **43 + 24 =** 40 + 20 + 3 + 4 = 60 + 7 = 67 Or, preferably
	As soon as possible, refine this method to a much quicker and clearer '**Partition Jot'** approach 	
	As before, develop these methods, especially Partition Jot, towards crossing the 10s and then 100s.	
	 	 
	This method will soon become the recognised jotting to support the teaching of partitioning. It can be easily extended to 3 and even 4 digit numbers when appropriate.	For certain children, the traditional partitioning method can still be used for 3 digit numbers, but is quite laborious for 4 digit numbers.
Y3/4	 	
	Partition jot is also extremely effective as a quicker alternative to column addition for decimals.	Some simple decimal calculations can also be completed this way.
Y5/6	 	
	For children with higher-level decimal place value skills, partition jot can be used with more complex decimal calculations or money.	





Stage 3 — Expanded Method in Columns

Y3

Column methods of addition are introduced in Year 3, but it is crucial that children still see mental calculation as their first principle, especially for 2 digit numbers.

Column methods should only be used for more difficult calculations, usually with 3 digit numbers that cross the thousands boundary or most calculations involving 4 digit numbers and above.

N.B. Even when dealing with bigger numbers / decimals, children should still look for the opportunity to calculate mentally (E.g. 4675 + 1998)

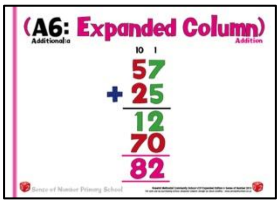
2 digit examples are used below simply to introduce column methods to the children. Most children would continue to answer these calculations mentally or using a simple jotting.

Using the column, children need to learn the principle of adding the ones first rather than the tens.

The ‘expanded’ method is a very effective introduction to column addition. It continues to use the partitioning strategy that the children are already familiar with, but begins to set out calculations vertically. It is particularly helpful for automatically ‘dealing’ with the ‘carry’ digit.

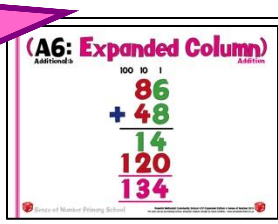
Y3/4

A. Single ‘carry’ in ones

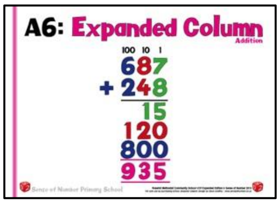


B. ‘Carry’ in ones and tens

‘Eighty plus forty equals one hundred and twenty, because ‘eight plus four equals twelve.



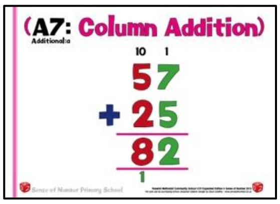
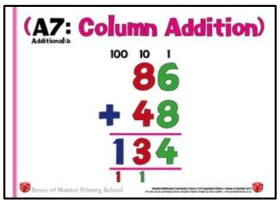
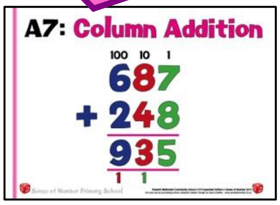
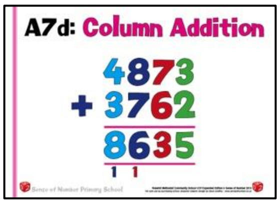
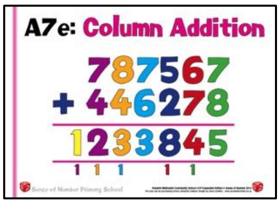
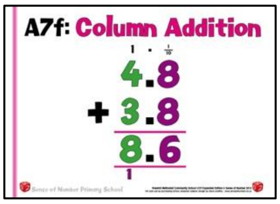
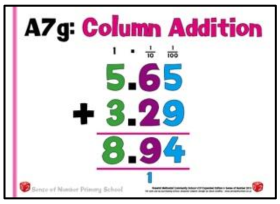
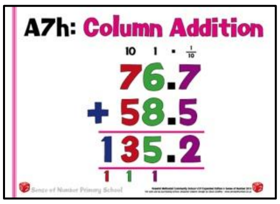
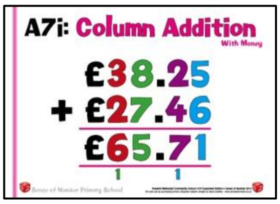
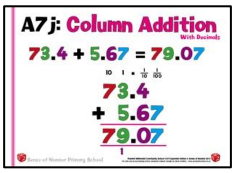
Once this method is understood, it can quickly be adapted to using with three digit numbers. It is rarely used for 4 digits and beyond as it becomes too unwieldy.



The time spent on practising the expanded method will depend on security of number facts recall and understanding of place value.

Once the children have had enough experience in using expanded addition, and have also used practical resources (Base 10 / place value counters) to model exchanging in columns, they can be taken on to standard, ‘traditional’ column addition.

Stage 4 — Column Method

Y3/4	As with the expanded method, begin with 2 digit numbers, simply to demonstrate the method, before moving to 3 digit numbers. Make it <u>very clear</u> to the children that they are still expected to deal with all 2 digit (and many 3 digit) calculations mentally (or with a jotting), and that the column method is designed for numbers that are too difficult to access using these ways. The column procedure <u>is not</u> intended for use with 2 digit numbers.
	'Carry' ones then ones and tens Use the words 'carry ten' and 'carry hundred', not 'carry one'
Record carry digits below the line.	  
Y4	Once confident, use with 4 digit numbers **(Year 4).**
	
Y5/6	Extend to 5/6 digit calculations then decimal calculations **(Year 5)**
If children make repeated errors at any stage, they can return to the expanded method or an earlier jotting.	    
	The key skill in upper Key Stage 2 that needs to be developed is the laying out of the column method for calculations with decimals in different places. 

Subtraction Progression

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact (e.g. 16 – 7), and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Counting Back (Taking away) ***Counting On***

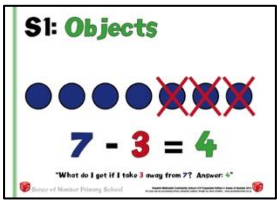
When should we count back and when should we count on?

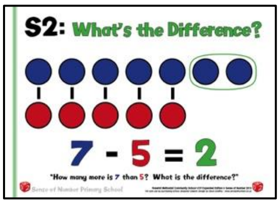
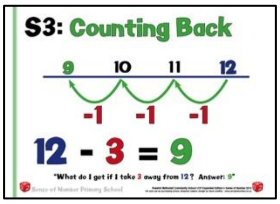
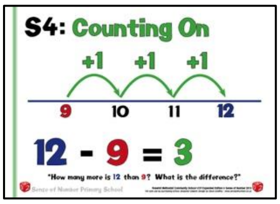
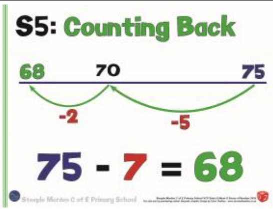
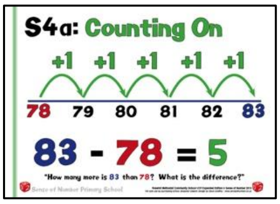
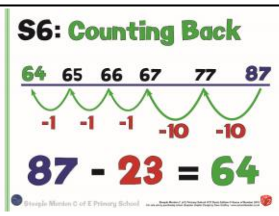
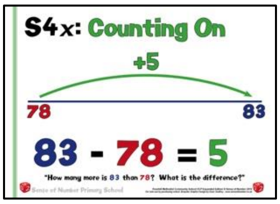
This will alter depending on the calculation, but often the following rules apply;

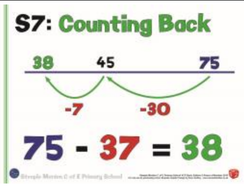
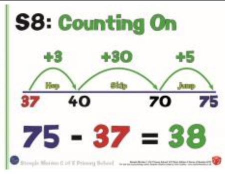
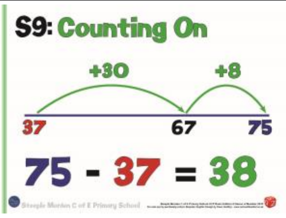
If the numbers are far apart, or there isn't much to subtract (278 – 24) then count back.

If the numbers are close together (206 – 188), then count up.

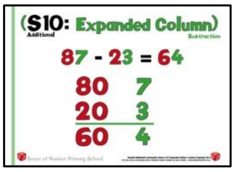
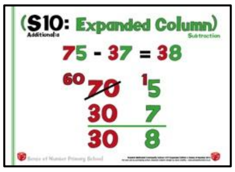
In many cases, either strategy would be suitable, depending on preference (743 – 476).

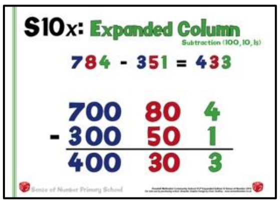
INTRO	Subtraction by counting back (or taking away)	Subtraction by counting up (or complementary addition)
FS/Y1	Early subtraction in EYFS will primarily be concerned with '***taking away***', and will be modelled using a wide range of models and resources.	
		
	This will continue in Year 1 using practical resources to 'take away' and then counting back using images such as a number line.	In Year 1, it is also vital that children understand the concept of subtraction as ***'finding a difference'*** and realise that **<u>any</u>** subtraction can be answered in 2 different ways, either by counting up or counting back. Again, this needs to be modelled and consolidated regularly using a wide range of resources, especially multilink towers, counters and Numicon.

		
Stage 1	**Using the Empty Number Line**	
	Subtraction by counting back (or taking away)	**Subtraction by counting up (or complementary addition)**
	The empty number line helps to record or explain the steps in mental subtraction. It is an ideal model for **counting back** and **bridging ten**, as the steps can be shown clearly. It can also show **counting up** from the smaller to the larger number to **find the difference.**	
Y1	The steps often bridge through a multiple of 10. **12 – 3 = 9**	Small differences can be found by counting up **12 – 9 = 3**
		
Y2/3	This is developed into crossing any multiple of 10 boundary. **75 – 7 = 68**	For 2 (or 3) digit numbers close together, count up **83 – 78 = 5** First, count in ones
		
	For 2 digit numbers, count back in 10s and 1s **87 – 23 = 64**	Then, use number facts to count in a single jump
		
	Then subtract tens and ones in single jumps (87 – 20 – 3)	Continue to spot small differences with 3 digit numbers **(403 – 397 = 6)**
	Some numbers (75 – 37) can be subtracted just as quickly either way.	
	Either count back 30 then count back 7	Or count up from smaller to the larger number, initially with a 'triple jump' strategy of jumping to the next 10, then multiples of 10, then to the target number.

<table>
<tr><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>Some children prefer to jump in tens and ones, which is an equally valid strategy, as it links to the mental skill of 'counting up from any number in tens'</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td rowspan="2">Stage 2</td><td colspan="2">Expanded Method & Number Lines (continued)</td></tr>
<tr><td>Subtraction by counting back
Expanded Method</td><td>Subtraction by counting up
Number Lines (continued)</td></tr>
<tr><td></td><td colspan="2">In Year 3 children are expected to be able to use both jottings <u>and</u> written column methods to deal with 3 digit subtractions.
It is very important that they have had regular opportunities to use the number line 'counting up' approach first so that they already have a secure method that is almost their first principle for most 2 and 3 digit subtractions.
This means that once they have been introduced to the column method they have an alternative approach that is often preferable, depending upon the numbers involved.
The number line method also gives those children who can't remember or successfully apply the column method an approach that will work with any numbers (even 4 digit numbers and decimals) if needed.</td></tr>
<tr><td>Y3/4</td><td>The expanded method of subtraction is an excellent way to introduce the column approach as it maintains the place value and is much easier to model practically with place value equipment such as Base 10 or place value counters

Introduce the expanded method with 2 digit numbers, but only to explain the process. Column methods are very rarely needed for 2 digit calculations.

Partition both numbers into tens and ones, firstly with no exchange then exchanging from tens to the ones.
87 – 23 75 – 37
</td><td></td></tr>
<tr><td>A</td><td>Take the method into three digit numbers.
Subtract the ones, then the tens, then the hundreds.
Demonstrate without exchanging first.
784 – 351</td><td></td></tr>
</table>



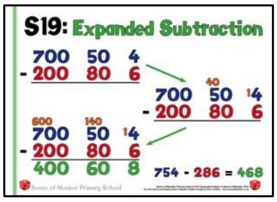
For examples without exchanging, the number line method takes considerably longer than mental partitioning or expanded.

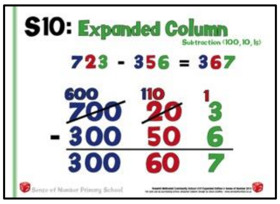
B

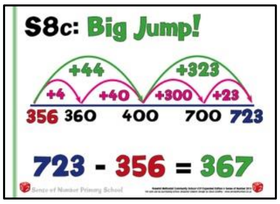
Move towards exchanging from hundreds to tens and tens to ones, in two stages if necessary

723 – 356

The example below shows 2 alternatives, for children who need different levels of support from the image.

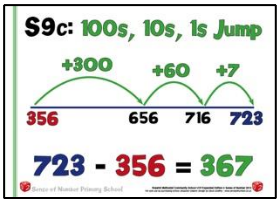






As before, many children prefer to count in hundreds, then tens, then ones.

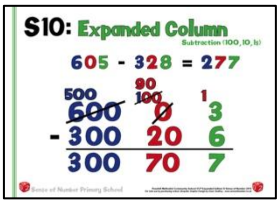
Eg, where exchanging is needed, then the number line method is equally as efficient, and is often easier to complete.

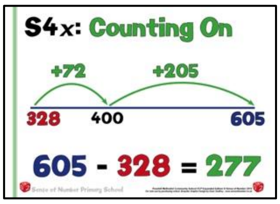


C

Use some examples which include the use of zeros e.g. **605 – 328**.

For numbers containing zeros, counting up is often the most reliable method.

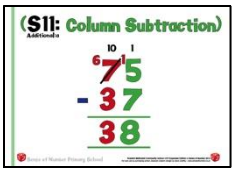


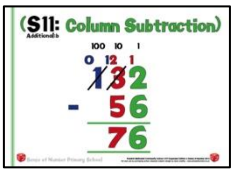


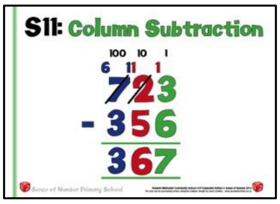
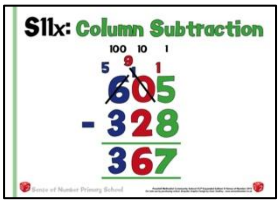
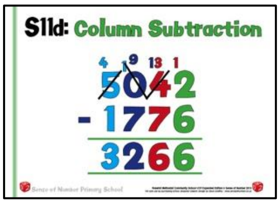
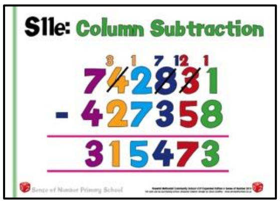
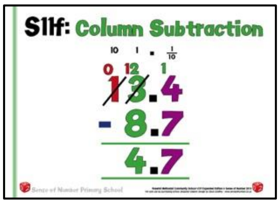
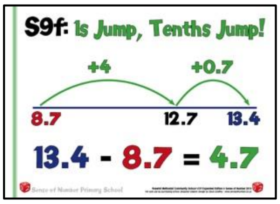
Continue to use expanded subtraction until both number facts and place value are considered to be very secure!

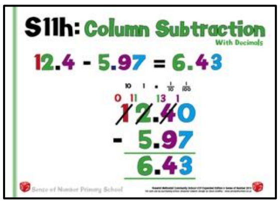
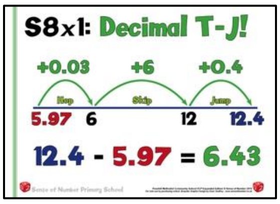
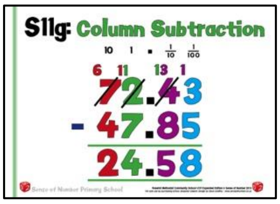
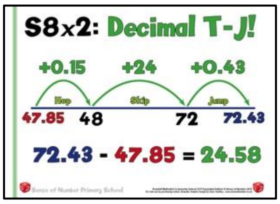
Stage 3 — Standard Column Method (Decomposition)

	Subtraction by counting back Standard Method	Subtraction by counting up Number Lines (continued)
Mainly **Y4+**	**Decomposition relies on secure understanding of the expanded method, and simply displays the same numbers in a contracted form.** As with the expanded method, and using practical resources such as place value counters to support the teaching, children in Year 4 will quickly move from decomposition via 2-digit number 'starter' examples to 2 / 3 digit and then 3 digit columns. **75 – 37** **132 – 56**	





<table>
<tr><td></td><td>723 – 356

</td><td>Continue to refer to digits by their actual value, not their digit value, when explaining a calculation. E.g. One hundred and twenty subtract fifty.</td></tr>
<tr><td></td><td>Again, use examples containing zeros, remembering that it may be easier to count on with these numbers (see Stage 2)
605 – 328

</td><td></td></tr>
<tr><td>Y4</td><td>From late Y4 onwards, move onto examples using 4 digit (or larger) numbers and then onto decimal calculations.
5042 – 1776

</td><td></td></tr>
<tr><td>Y5/6</td><td>In Years 5 & 6 apply to any ‘big number’ examples.</td><td></td></tr>
<tr><td></td><td>
</td><td></td></tr>
<tr><td></td><td colspan="2">Both methods can be used with decimals, although the counting up method becomes less efficient and reliable when calculating with more than two decimal places.</td></tr>
<tr><td></td><td>13.4 – 8.7</td><td>13.4 – 8.7</td></tr>
<tr><td></td><td>
</td><td>
</td></tr>
<tr><td></td><td>12.4 – 5.97</td><td>12.4 – 5.97</td></tr>
</table>

	 S11h: Column Subtraction With Decimals 12.4 - 5.97 = 6.43 12.40 - 5.97 6.43	 S8x1: Decimal T-J! +0.03 +6 +0.4 5.97 6 12 12.4 12.4 - 5.97 = 6.43
	72.43 – 47.85	
	 S11g: Column Subtraction 72.43 - 47.85 24.58	 S8x2: Decimal T-J! +0.15 +24 +0.43 47.85 48 72 72.43 72.43 - 47.85 = 24.58

Multiplication Progression

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

These notes show the stages in building up to using an efficient method for

- two-digit by one-digit multiplication by the end of Year 3,
- three-digit by one-digit multiplication by the end of Year 4,
- four-digit by one-digit multiplication and two/three-digit by two-digit multiplication by the end of Year 5
- three/four-digit by two-digit multiplication and multiplying 1-digit numbers with up to 2 decimal places by whole numbers by the end of Year 6.

To multiply successfully, children need to be able to:

- recall all multiplication facts to 12 × 12;
- partition numbers into multiples of one hundred, ten and one;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- similarly apply their knowledge to simple decimal multiplications such as 0.7 x 5, 0.7 x 0.5, 7 x 0.05, 0.7 x 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

Note:

Children need to acquire one efficient written method of calculation for multiplication, which they know they can rely on when mental methods are not appropriate.

It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

These mental methods are often more efficient than written methods when multiplying.

Use partitioning and grid methods until number facts and place value are secure.

For a calculation such as 25 x 24, a quicker method would be 'there are four 25s in 100 so 25 x 24 = 100 x 6 = 600

When multiplying a 3/4 digit x 2-digit number the standard method is usually the most efficient.

At all stages, use known facts to find other facts.

E.g. Find 7 x 8 by using 5 x 8 (40) and 2 x 8 (16).

Mental Multiplication

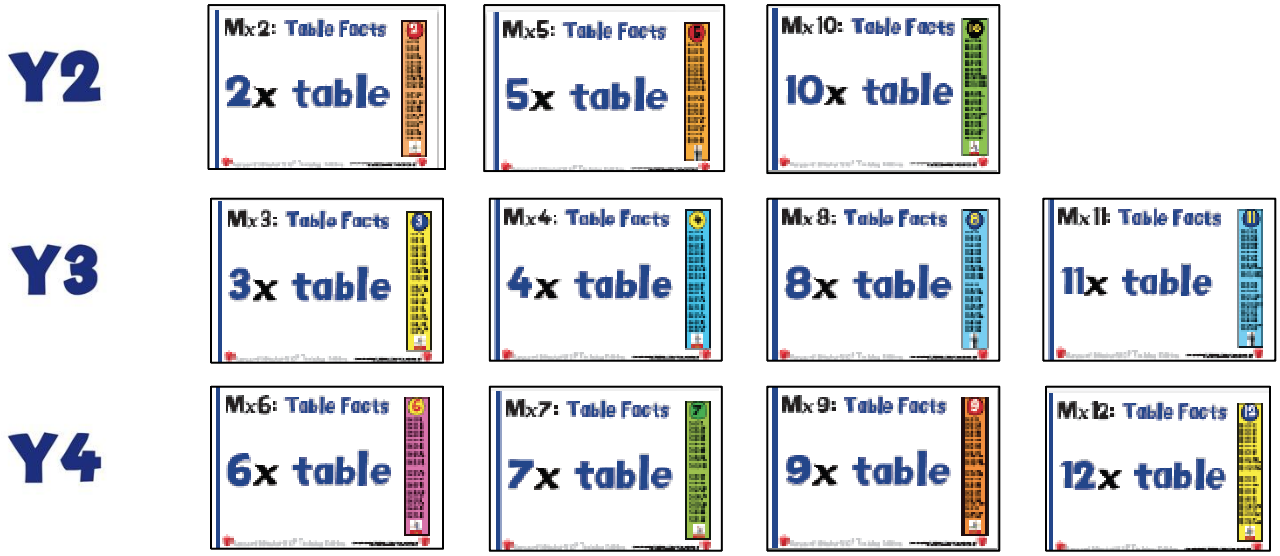
In a similar way to addition, multiplication has a range of mental strategies that need to be developed both before and then alongside written methods (both informal and formal).

Tables Facts

In Key Stage 2, however, before any written methods can be securely understood, children need to have a bank of multiplication tables facts at their disposal, which can be recalled instantly.

The learning of tables facts does begin with counting up in different steps, but by the end of Year 4 it is expected that most children can instantly recall all facts up to 12 x 12.

The progression in facts is as follows:-



Once the children have established a bank of facts, they are ready to be introduced to jottings and eventually written methods.

Doubles & Halves

The other facts that children need to know by heart are doubles and halves, eg, double the 4 times table to get the 8 times table, double again for the 16 times table etc

As a general guidance, children should know the following doubles: -

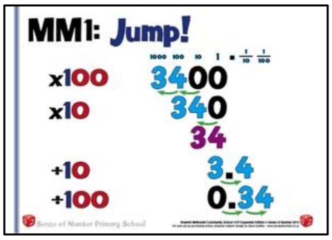
Year Group	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Doubles and Halves	All doubles and halves from double 1 to double 10 / half of 2 to half of 20	All doubles and halves from double 1 to double 20 / half of 2 to half of 40 (eg, double 17=34, half of 28 = 14)	Doubles of all numbers to 100 with units digits 5 or less, and corresponding halves (E.g. Double 43, double 72, half of 46) Reinforce doubles & halves of all multiples of 10 & 100 (eg, double 800, half of 140)	Addition doubles of numbers 1 to 100 (eg,. 38 + 38, 76 + 76) and their corresponding halves Revise doubles of multiples of 10 and 100 and corresponding halves	Doubles and halves of decimals to 10 – 1 d.p. (eg, double 3.4, half of 5.6)	Doubles and halves of decimals to 100 – 2 d.p. (eg, double 18.45, half of 6.48)

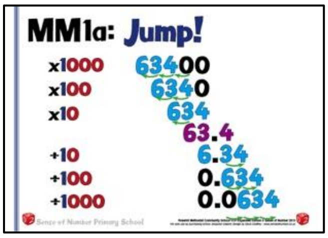
Before certain doubles / halves can be recalled, children can use a simple jotting to help them record their steps towards working out a double or half.

Multiplying by 10 / 100 / 1000

The first strategy is usually part of the Year 5 & 6 teaching programme for decimals, namely that digits move to the left when multiplying by 10, 100 or 1000, and to the right when dividing.

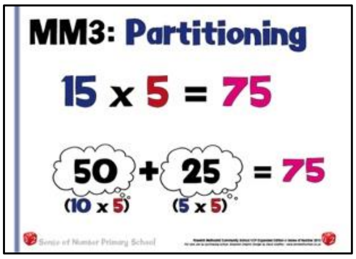
This also secures place value by emphasising that the decimal point doesn't ever move, and that the digits move around the decimal point (not the other way round!)

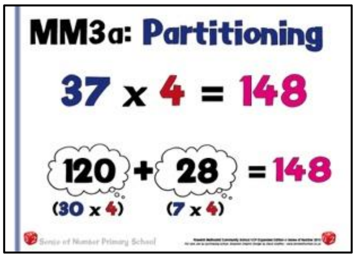




Partitioning is an equally valuable strategy for multiplication, and can be quickly developed from a jotting to a method completed entirely mentally. It is clearly linked to the grid method of multiplication, but is taught as a 'partition jot' so that children, by the end of Year 4, have become skilled in mentally partitioning 2 and 3 digit numbers when multiplying (with jottings when needed).

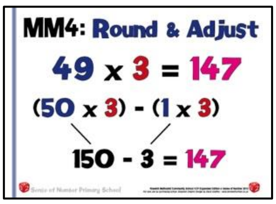
By the end of Year 6 children should be able to mentally partition most simple 2 & 3 digit, and also decimal multiplications.

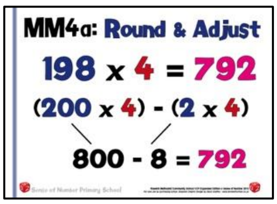


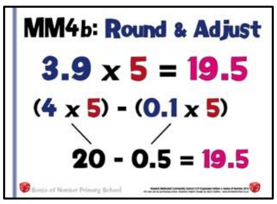


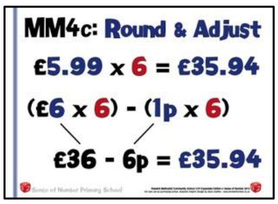
Round & Adjust is also a high quality mental strategy for multiplication, especially when dealing with money problems in Upper KS2. Once children are totally secure with rounding and adjusting in addition, they are shown how the strategy extends into multiplication, where they round then adjust by the multiplier.

eg, For 39 x 6 round to 40 x 6 (240) then adjust by 1 x 6 (6) to give a product of 240 – 6 = 234.





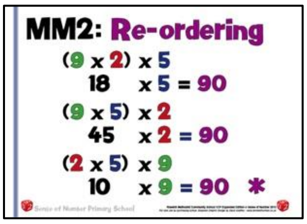


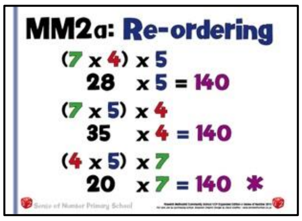


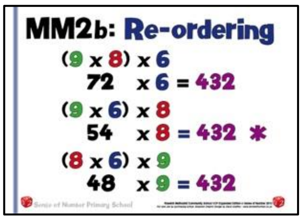
Y4 Y4/5 Y5 Y5/6

Re-ordering is similar to **Number Bonds** in that the numbers are calculated in a different order - the children look at the numbers that need to be multiplied and using commutativity, rearrange them so that the calculation is easier.

The asterisked calculation in each of the examples below is probably the easiest / most efficient rearrangement of the numbers.

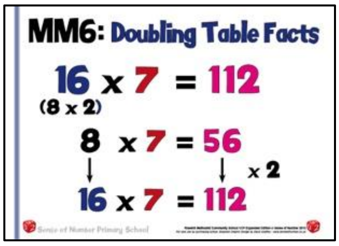




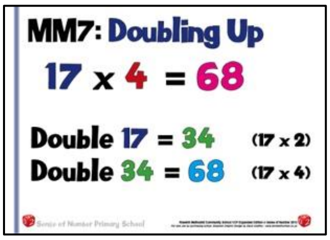


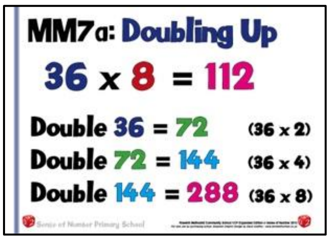
<u>Doubling</u> strategies are probably the most crucial of the mental strategies for multiplication, as they can make difficult long multiplication calculations considerably simpler.

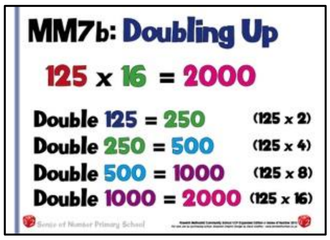
Initially, children are taught to double one table to find another, eg, doubling the 3s to get the 6s. This can then be applied to any table: -



<u>Doubling Up</u> enables multiples of **4, 8** and **16** onwards to be calculated by constant doubling: -

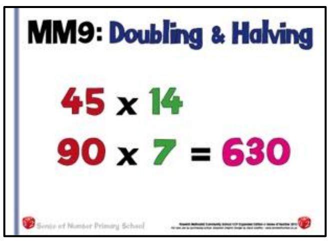


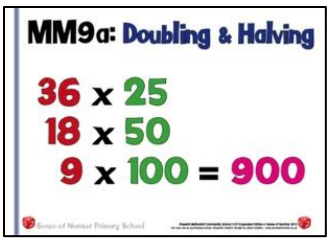


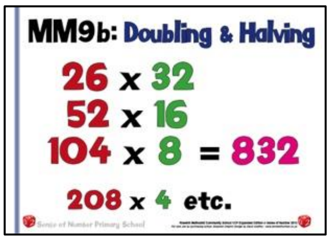


<u>Doubling & Halving</u> is probably the best strategy available for simplifying a calculation.

Follow the general rule that if you double one number within a multiplication, and halve the other number, then the product stays the same.

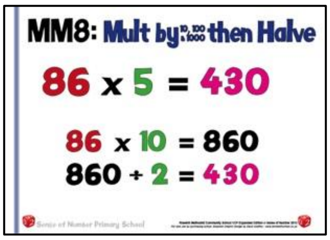


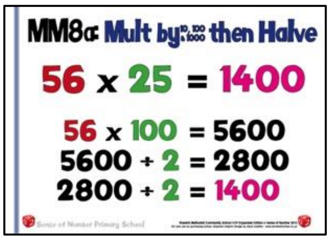




<u>Multiplying by 10 / 100 / 1000 and then Halving</u>

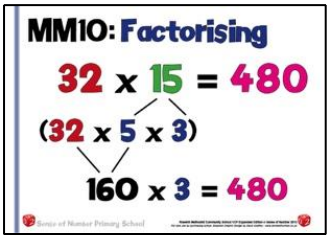
The final doubling / halving strategy works on the principle that multiplying by 10 / 100 is straightforward, and this can enable you to easily multiply by 5, 50 or 25.

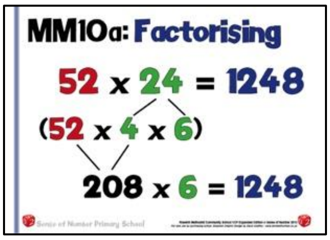




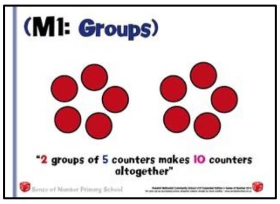
Factorising

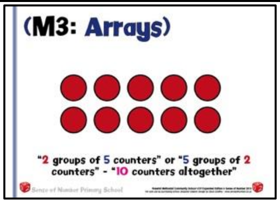
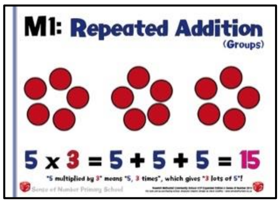
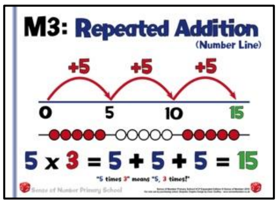
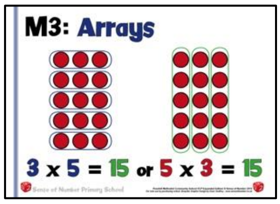
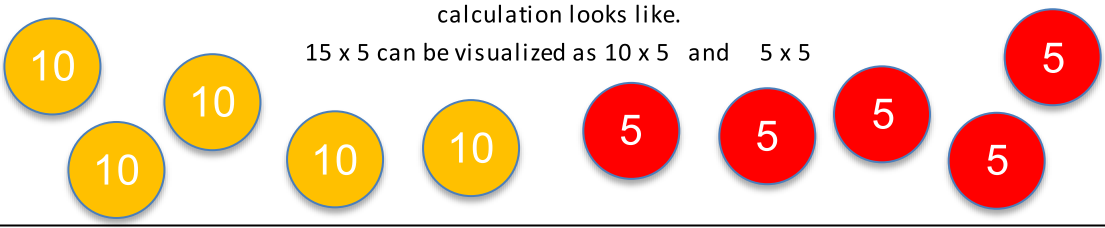
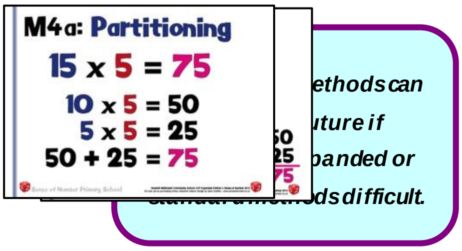
The only remaining mental strategy, which again can simplify a calculation, is factorising. Multiplying a 2-digit number by 36, for example, may be easier if multiplying by a factor pair of 36 (x6 then x6, or x9 then x4, even x12 then x3).





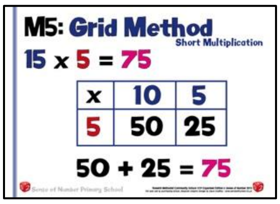
Written Multiplication

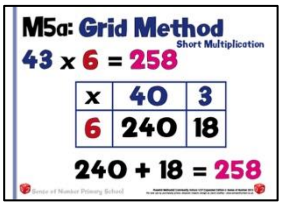
Stage 1	Number Lines, Arrays & Mental Methods
FS	In Early Years, children are introduced to grouping, and are given regular opportunities to put objects into groups of 2, 3, 4, 5 and 10. They also stand in different sized groups, and use the term 'pairs' to represent groups of 2. Using resources such as Base 10 apparatus, Numicon, multi-link or an abacus, children visualise counting in ones, twos, fives and tens, saying the multiples as they count the pieces, eg, Saying '10, 20, 30' or 'Ten, 2 tens, 3 tens' whilst counting Base 10 pieces
Y1	Begin by introducing the concept of multiplication as repeated addition. Children will make and draw objects in groups (again using resources such as Numicon, counters and multi-link), giving the product by counting up in 2s, 5s, 10s and beyond and writing the multiplication statement.  Extend into making multiplication statements for 3s and 4s, using resources (especially real life equipment such as cups, cakes, sweets etc.) Make sure from the start that all children say the multiplication fact the correct way round, using the word 'multiply' more often than the word 'times'. For the example above, there are 5 counters in 2 groups, showing 5 multiplied by 2 (5x2), not 2 times 5. It is the '5' which is being scaled up / made bigger / multiplied. '5 multiplied by 2' shows '2 groups of 5' or 'Two fives'
	Develop the use of the array to show linked facts (commutativity). Emphasise that all multiplications can be worked out either way. (2 x 5 = 5 x 2 = 10)

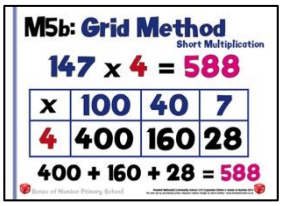
	
Y2	Build on children's understanding that multiplication is repeated addition, using arrays and number lines to support the thinking. Explore arrays in real life.  Start to emphasise commutativity, e.g. that 5 x 3 = 3 x 5
Continue to emphasise multiplication the correct way round. *E.g. 5 x 3 = 5 + 5 + 5* *5 multiplied by 3 = 15*	  
Y3	Extend the above methods to include the 3, 4, 6 and 8 times tables. Continue to model calculations, where appropriate, with resources such as Numicon, Place Value Counters or bead strings, counting quickly in different steps and placing / moving the resource. Extend the use of resources to 2 digit x 1 digit calculations so that children can visualize what the calculation looks like. 15 x 5 can be visualized as 10 x 5 and 5 x 5 
	Then begin to partition using jottings and number lines. 
	Extend the methods above to calculations which give products greater than 100.

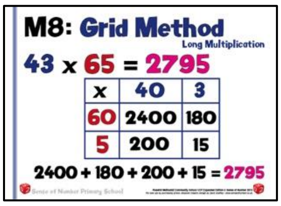
Grid Method

The Grid Method develops a clear understanding of place value as well as being an efficient method, and is especially useful in Years 4 and 5.

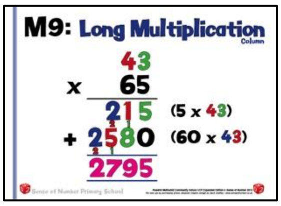




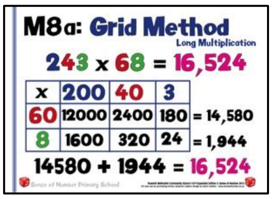


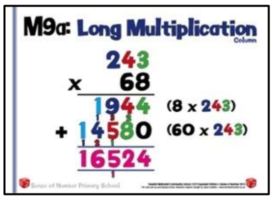


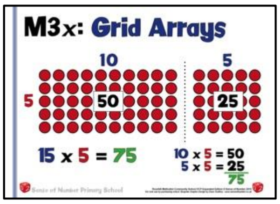
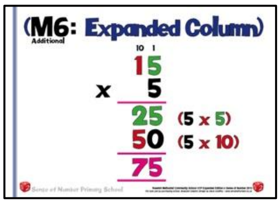
Column procedures still retain some element of place value, but, particularly for long multiplication, tend to rely on memorising a 'method', and can lead to many children making errors with the method (which order to multiply the digits, when to 'add the zero', dealing with the 'carry' digits etc.) rather than the actual calculation.



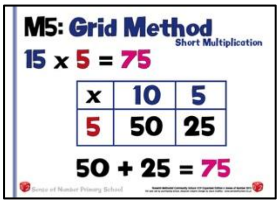
Once the calculations become more unwieldy (4 digit x 1 digit or 3 / 4 digit x 2 digit) then grid method begins to lose its effectiveness, as there are too many zeroes and part products to deal with. At this stage column procedures are far easier, and, once learned, can be applied much quicker. Grid methods can still be used by some pupils who find columns difficult to remember, and who regularly make errors, but children should be encouraged to move towards columns for more complex calculations.



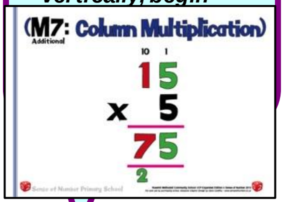


<table>
<tr><th rowspan="2">Stage 2</th><th colspan="2">Written Methods - Short Multiplication</th></tr>
<tr><th>Grid Multiplication
(Mental 'Jotting')</th><th>Column multiplication
(Expanded method into standard)</th></tr>
<tr><td></td><td>The grid method of multiplication is a simple, alternative way of recording the jottings shown previously.
If necessary (for some children) it can initially be taught using an array to show the actual product.</td><td>The expanded method links the grid method to the standard method.
It still relies on partitioning the tens and ones, but sets out the products vertically.
Children will use the expanded method until they can securely use and explain the standard method.</td></tr>
<tr><td></td><td>
</td><td>
</td></tr>
<tr><td>Y3</td><td>It is recommended that the grid method is used as the main method within Year 3. It clearly maintains place value, and helps children to visualise and understand the calculation better.</td><td>Later in the year, the column method is introduced and children are given the choice of using either grid or standard.</td></tr>
</table>

Place the 'carry' digit below the line



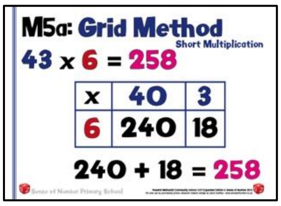
calculations vertically, begin

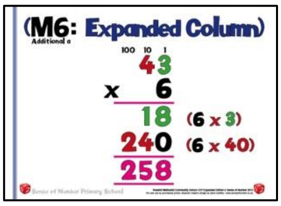


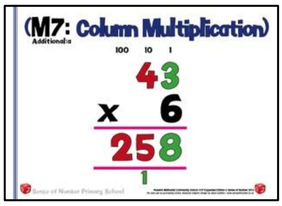
Y4

Continue to use both grid and column methods in Year 4 for more difficult 2 digit x 1 digit calculations, extending the use of the grid method into mental partitioning for those children who can use the method this way.

At this point, the expanded method can still be used when necessary (to help 'bridge' grid with column), but children should be encouraged to use their favoured method (grid or column) whenever possible.







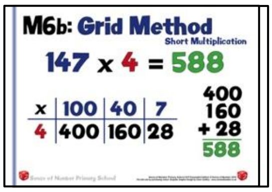
For 3 digit x 1 digit calcualtions, both grid and standard methods are efficient.

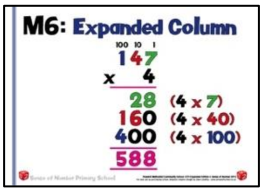
Continue to use the grid method to aid place value and mental arithmetic.

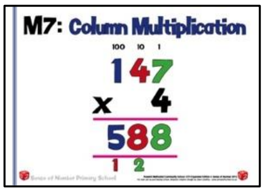
Develop the column method for speed, and to make the transition to long multiplication easier.

Children in Year 4 will have a clear choice of 2 secure methods, and will be able to develop both accuracy and speed in multiplication.

If children find it difficult to add the 'part products' then set them out vertically (or encourage column method).

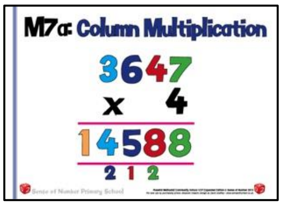






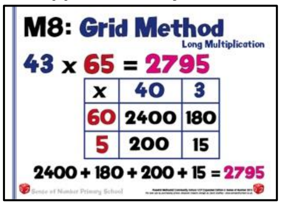
Y5

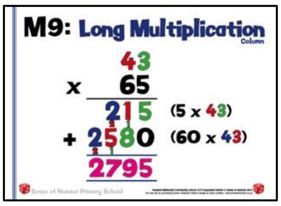
For a 4 digit x 1 digit calculation, the column method, once mastered, is quicker and less prone to error. The grid method may continue to be the main method used by children who find it difficult to remember the column procedure, or children who need the visual link to place value.



Stage 3	Long Multiplication (2-digit x 2-digit)	
	Grid Multiplication	Column multiplication (Expanded method into standard)

Y5	Extend the grid method to 2-digit × 2-digit, asking children to estimate first so that they have a general idea of the answer. ***(43 × 65 is approximately 40 × 70 = 2800.)***	**Children should only use the 'standard' column method of long multiplication if they can regularly get the correct answer using this method.**





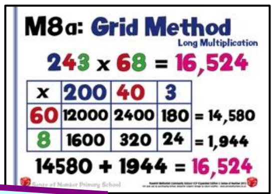
As mentioned earlier, the grid method is often the 'choice' of many children in Years 5 and 6, due to its ease in both procedure and understanding / place value and is the method that children will mainly use for simple long multiplication calculations.

There is no 'rule' regarding the position of the 'carry'digits. Each choice has advantages and complications.

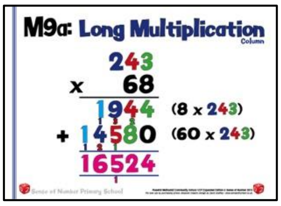
Either carry the digits mentally or have your own favoured position for these digits.

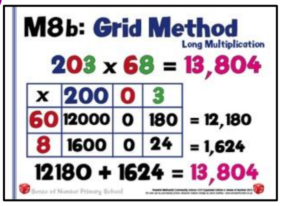
Y6	For 3 digit x 2 digit calculations, grid method is quite inefficient, and has much scope for error due to the number of 'part-products' that need to be added. **Use this method when you find the standard method to be unreliable or difficult to remember.**	**Most children, at this point, should be encouraged to choose the standard method.** For 3 digit x 2 digit calculations it is especially efficient, and less prone to errors when mastered. **Although children may find the grid method easier to apply, it is much slower / less efficient.**

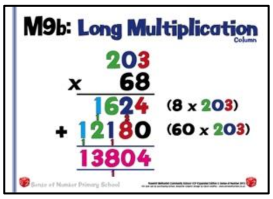
Again, estimate first:
243 × 68 is approximately 200 × 70 = 14000.



Add these numbers for the overall product.

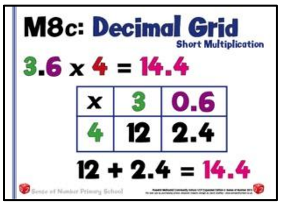


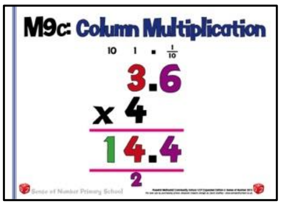




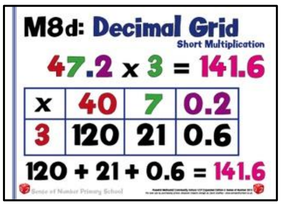
Many children will find the use of Grid method as an efficient method for multiplying decimals.

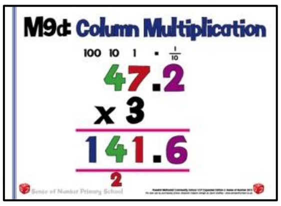
Extend the use of standard method into the use of decimals.





Y6





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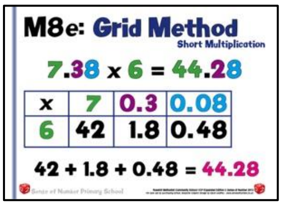
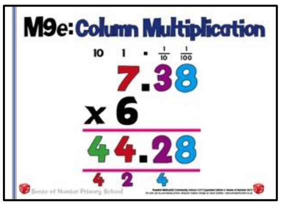
<tr><td></td><td>
</td><td>
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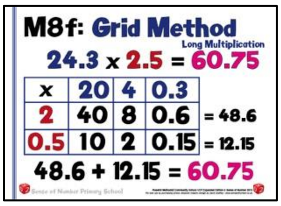
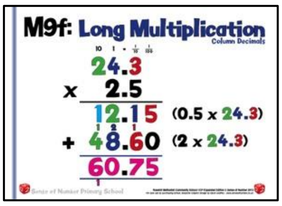
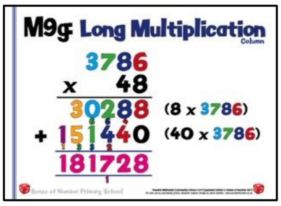
<tr><td></td><td>
</td><td>
</td></tr>
<tr><td></td><td></td><td>By this time children meet 4 digits by 2 digits, the only efficient method is the standard method for Long Multiplication.</td></tr>
<tr><td></td><td></td><td>
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Division Progression

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence.

These notes show the stages in building up to long division through Years 3 to 6 – first using short division 2 digits ÷ 1 digit, extending to 3 / 4 digits ÷ 1 digit, then long division 4 / 5 digits ÷ 2 digits.

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- recall multiplication and division facts to 12 × 12, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

Children need to acquire one efficient written method of calculation for division, which they know they can rely on when mental methods are not appropriate.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out expanded and standard written methods of division successful, children also need to be able to:

- visualise how to calculate the quotient by visualising repeated addition;
- estimate how many times one number divides into another – for example, approximately how many sixes there are in 99, or how many 23s there are in 100;

- multiply a two-digit number by a single-digit number mentally;
- understand and use the relationship between single digit multiplication, and multiplying by a multiple of 10. (e.g. 4 x 7 = 28 so 4 x 70 = 280 or 40 x 7 = 280 or 4 x 700 = 2800.)
- subtract numbers using the column method (if using 'chunking')

The above points are crucial. If children do not have a secure understanding of these prior-learning objectives then they are unlikely to divide with confidence or success, especially when attempting the 'chunking' method of division.

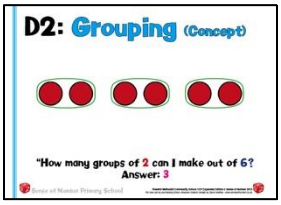
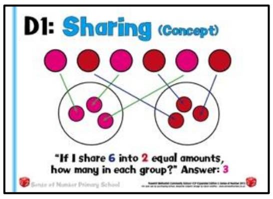
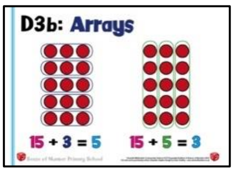
For example, without a clear understanding that 72 can be partitioned into 60 and 12, 40 and 32 or 30 and 42 (as well as 70 and 2), it would be difficult to divide 72 by 6, 4 or 3 using the 'chunking' method.

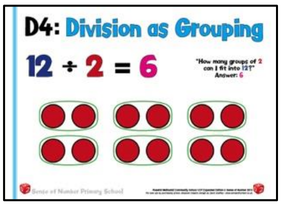
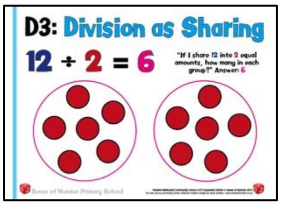
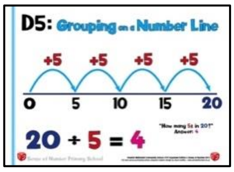
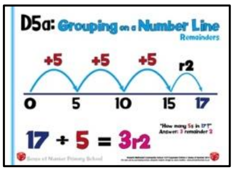
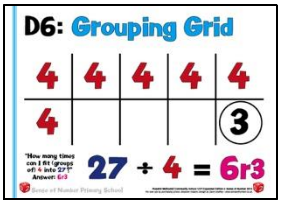
72 ÷ 6 'chunks' into 60 and 12

72 ÷ 4 'chunks' into 40 and 32

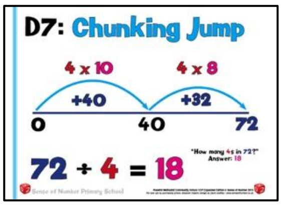
72 ÷ 3 'chunks' into 30 and 42 (or 30, 30 and 12)

Please note that there are two different 'policies' for chunking. At Steeple Morden we have made the decision to teach chunking as a mental arithmetic / number line process, and prefer to count forwards in chunks rather than backwards.

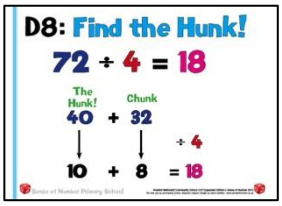
Stage 1	Concepts and Number Lines (pre-chunking)	
	Grouping	Sharing
FS	From EYFS onwards, children need to explore practically both **grouping** and **sharing**. Links can then be made in both KS1 and KS2 between sharing and fractions.	
Y1	Begin by giving children opportunities to use concrete objects, pictorial representations and arrays with the support of the teacher. Use the words 'sharing' and 'grouping' to identify the concepts involved. Identify the link between multiplication and division using the array image.	
		
	 	
Y2	Identify Grouping as the key model for division. Relate to knowledge of multiplication facts. Use the key vocabulary: '20 ÷ 5 means how many 5's can I fit into 20?'	Identify Sharing as the secondary model of division.

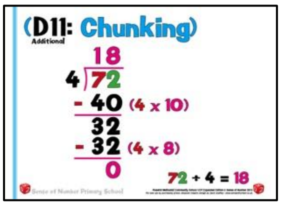
		
	Counting on is the easiest route when using a number line to solve a division calculation.	
	 	
Y3	Continue to give children practical images for division by grouping: e.g. use PE mats and ask children to move into groups of 4. The remainder go into a hoop. Use Numicon shapes – how many 4 pieces can I fit into 27 (made of two tens and a seven piece).	***Regularly stress the link between multiplication and division, and how children can use their tables facts to divide by counting forwards in steps.***
		

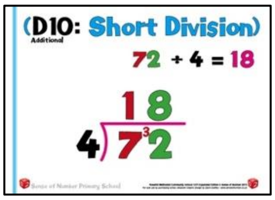
Stage 2 — Chunking & Standard Methods

	Chunking	Standard Methods
	As previously encountered in Y2, developing an understanding of division with the number line is an excellent way of linking division to multiplication. It can show division both as repeated subtraction, but it is simpler to show division by counting forward to find how many times one number 'goes into' another.	
Y3		
		The key method in Year 3 is the use of number lines and the mental 'chunking' method. The Short Division ('Bus Stop') method is introduced in Year 4 using the '**sharing model**' /Base 10. The calculation starts with, 'I have 7 tens, to share between 4 people. That's 1 each with 3 remaining. These three tens are exchanged into 30, ones. The 32 ones are now shared between 4 people – that's 8, ones each.'

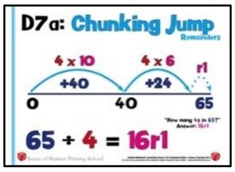
Y4

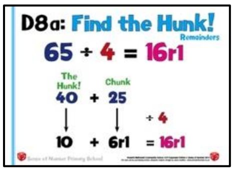


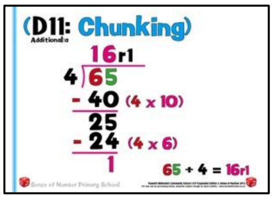


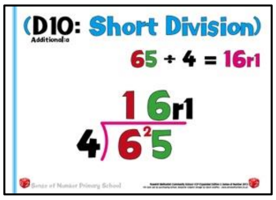


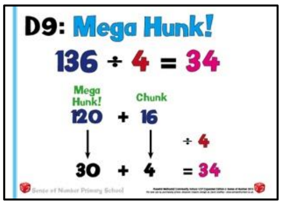
Children see chunking where the quotient includes remainders.

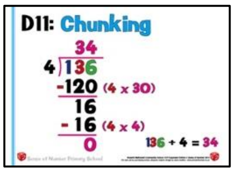


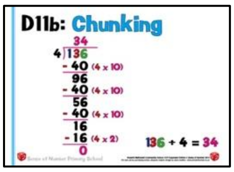


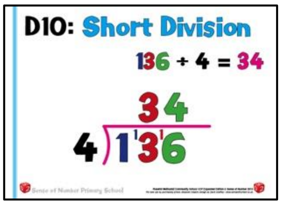




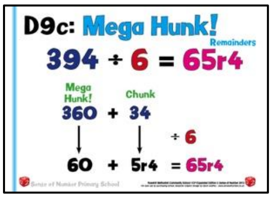


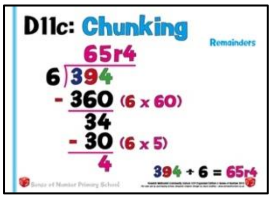


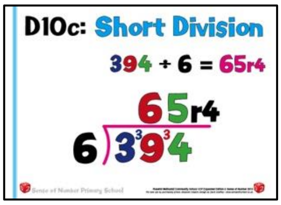


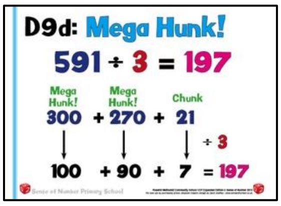


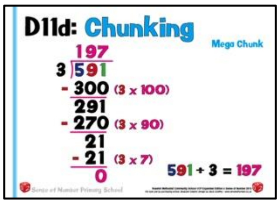
Y5

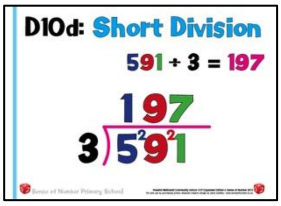


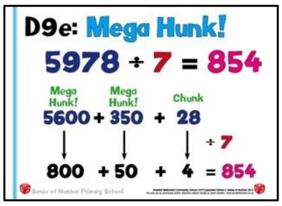


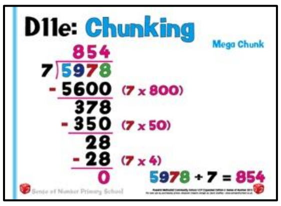


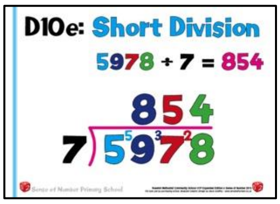












Begin by subtracting several chunks, but then try to find the biggest chunks of the divisor that can be subtracted.

Children should develop the ability to represent the quotient to include a straightforward remainder, but also as a decimal or fractional remainder.

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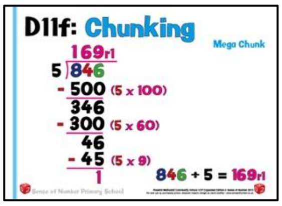
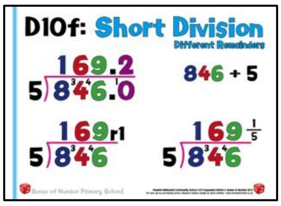
<tr><td></td><td>
</td><td>
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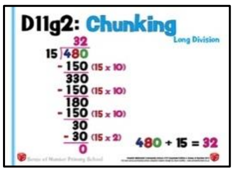
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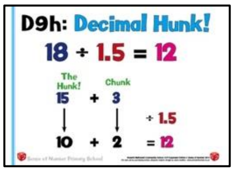
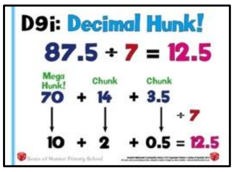
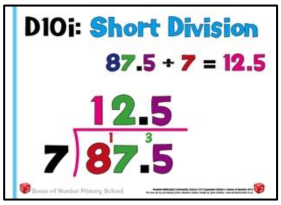
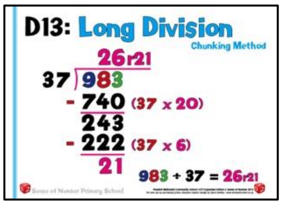
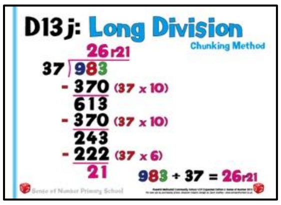
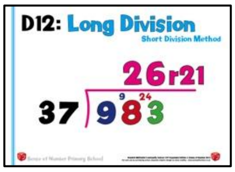
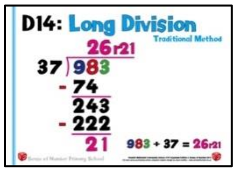
<tr><td>Y6</td><td>


</td><td>
</td></tr>
<tr><td></td><td>There are three different ways of calculating using Long Division: The Short Division method, the Traditional Method and the Chunking method. The Traditional Long Division method ignores place value and therefore is not considered to be as helpful as the Chunking Method, which is now the recommended strategy.</td><td></td></tr>
<tr><td></td><td>


</td><td>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</table>

If you wish to discuss any aspect of Maths in school please speak to your child's class teacher in the first instance.
I hope you will find this booklet useful in understanding how we teach calculation in school.

Jo Daniels
Maths Subject Leader

March 2018

MATHS IS FUN!

We know that many children are really enjoying using Mathletics as a fun way of practising Maths skills that we have learnt in school.

We currently introduce Mathletics in Year 2 in the classroom and encourage the children to 'have a go' at home. Children can then follow Mathletics all the way to Year 6!

Our new way of using the system is underway. Each class is set a weekly task on Mathletics and then after that has been completed, children are able to explore any further aspects of Maths they would like to. It is helpful to set tasks which link directly with a child's learning in school but it is also valuable for your child to 'play' and have a go at activities that they find particularly appealing.

If you haven't yet had a go please spend a few minutes looking at some of the fun activities. The learning platform is really easy to navigate around and it has been designed to give parents a clear indication of which aspects of Maths their child find simple and which they may need to work on! It will really be of benefit if you are able to sit with your child to get them going on the homework initially at least.

Here's how to get started:-

- Go to Mathletics.com/primary.
- Follow the link for pupils.
- Enter your child's username and password from the card that has been sent home from school.
- There are lots of fun activities and games to try.
- You can earn points and certificates.

Remember **red** / **amber** / **green**

Try your best to go green!

We know that many children will enjoy earning points, gold bars and entering into competition with their friends. The class teacher is able to see how each child is doing.

If children do earn certificates, these can be presented in our weekly Friday Celebration Assembly.

Mathletics not working on your home computer?

Some children and parents have been reporting problems with enabling flash on their computers at home. Flash is a program required to run activities fully on Mathletics. If you are having issues, then click this link to a tailored guide to show you how to enable flash specifically for Mathletics for the most popular web browsers.

http://www.3plearning.com/tech/flash/

unicef *Mathletics supports the work of Unicef in promoting an education for children throughout the world.*